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Quantifying Tolerance of a Nonlocal Multi-Qudit State to Any Local Noise

Elena R. Loubenets

Applied Mathematics Department, National Research University Higher School of Economics, Moscow 101000, Russia; elena.loubenets@hse.ru

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Abstract: We present a general approach for quantifying tolerance of a nonlocal N-partite state to any local noise under different classes of quantum correlation scenarios with arbitrary numbers of settings and outcomes at each site. This allows us to derive new precise bounds in d and N on noise tolerances for: (i) an arbitrary nonlocal N-qudit state; (ii) the N-qudit Greenberger–Horne–Zeilinger (GHZ) state; (iii) the N-qubit W state and the N-qubit Dicke states, and to analyse asymptotics of these precise bounds for large N and d.

Keywords: nonlocal N-qudit states; maximal Bell violation; tolerance to any local noise

1. Introduction

Nonlocality [1–3] of an N-qudit quantum state, *in the sense of its violation of a Bell inequality*, is a major resource for developing quantum information technologies. Conceptual and quantitative issues of Bell nonlocality in a general nonsignaling case have been analyzed in [4] and references therein. The main concepts and tools that were developed to describe and to study Bell nonlocality in a quantum case have been reviewed in [5]. (We further discuss only the notions of Bell nonlocality and locality and, therefore, mostly suppress the specification "Bell" before these terms.)

In quantum information applications, one, however, deals with noisy channels and, for a nonlocal N-qudit state $\rho_{d,N}$, $d \geq 2, N \geq 2$, it is important to evaluate amounts of noise not breaking the nonclassical character of its statistical correlations. Analytical and numerical bounds on the critical visibility of a nonlocal N-qudit state $\rho_{d,N}$ in a mixture *with white noise*:

$$(1-\beta)\,\frac{\mathbb{I}^{\otimes N}}{d^N} + \beta\rho_{d,N}, \quad \beta \in [0,1], \tag{1}$$

have been intensively studied in the literature: (i) for a nonlocal two-qudit state—in [6–10] and references therein; (ii) for some specific quantum correlation scenarios and specific N-qubit states—in [11–19]; and (iii) for an arbitrary nonlocal N-qudit state $\rho_{d,N}$, $N \geq 3, d \geq 3$—in [20].

However, precise analytical bounds on the critical visibility of a nonlocal N-qudit state $\rho_{d,N}$ in a mixture

$$(1-\beta)\,\zeta_{loc} + \beta\rho_{d,N}, \quad \beta \in [0,1], \tag{2}$$

with an arbitrary local noise (i.e. a noise described by a local N-qudit state ζ_{loc}) and, more generally, bounds on the *tolerance* of a nonlocal N-qudit state $\rho_{d,N}$ to any local noise are not, to our knowledge, known in a general N-qudit case, though, for a nonlocal family of joint probabilities under a bipartite ($N = 2$) correlation scenario, the similar concept—the resistance to noise—was introduced in [21] and further discussed in [5]. For the rigorous definition of the notion of the tolerance of a nonlocal state see Section 4.

We note that, for many quantum information applications based on Bell nonlocality, it is important to evaluate the maximal amount of noise tolerable by a nonlocal N-qudit state and this amount is determined specifically via the noise tolerance of a nonlocal state.

In the present paper, due to the general framework for Bell nonlocality developed in [4,22,23], we present a consistent approach to quantifying tolerance of a nonlocal N-partite quantum state to any local noise under different classes of quantum correlation scenarios with arbitrary numbers of settings and any spectral types of outcomes at each site. This allows us:

- to specify via parameters of an N-partite state the general analytical expressions for the noise tolerance of a nonlocal N-partite state (i) under $S_1 \times \cdots \times S_N$-setting quantum correlation scenarios with any number of outcomes at each site and (ii) under all quantum correlation scenarios with arbitrary numbers of settings and outcomes per site;
- to derive new precise lower/upper bounds in d and N on the noise tolerances and the maximal amounts of tolerable local noise for: (i) an arbitrary nonlocal N-qudit state; (ii) the N-qudit Greenberger-Horne-Zeilinger (GHZ) state; (iii) the N-qubit W state and the N-qubit Dicke states and to analyse asymptotics of these precise new bounds for large N and d.

2. General N-Partite Bell Inequalities

Let us shortly recall the notion of a general multipartite Bell inequality [24] with arbitrary numbers of settings and outcomes per site. For the general framework on the probabilistic description of an arbitrary multipartite correlation scenario with any number of settings and any spectral type of outcomes at each site, see [25].

Consider a correlation scenario, where each n-th of N parties performs $S_n \geq 1$ measurements with outcomes $\lambda_n \in [-1,1]$ and every measurement at n-th site is specified by a positive integer $s_n = 1, ..., S_n$. For concreteness, we label an $S_1 \times \cdots \times S_N$-setting scenario by $\mathcal{E}_S$, where $S = S_1 \times \cdots \times S_N$.

For a correlation scenario $\mathcal{E}_S$, denote by $P^{(\mathcal{E}_S)}_{(s_1,...,s_N)}$ the joint probability distribution of outcomes $(\lambda_1, \ldots, \lambda_N) \in [-1,1]^N$ under an N-partite joint measurement induced by measurements $s_1, ..., s_N$ at the corresponding sites and by

$$\begin{aligned} \mathcal{B}^{(\mathcal{E}_S)}_{\Phi_S} &= \sum_{s_1,...,s_N} \left\langle f_{(s_1,...,s_N)}(\lambda_1, \ldots, \lambda_N) \right\rangle_{\mathcal{E}_S}, \\ \Phi_S &= \{f_{(s_1,...,s_N)} : [-1,1]^N \to \mathbb{R} \mid s_n = 1, ..., S_n, \;\; n = 1, ..., N\}, \end{aligned} \tag{3}$$

a linear combination of averages (expectations)

$$\begin{aligned} &\left\langle f_{(s_1,...,s_N)}(\lambda_1, \ldots, \lambda_N) \right\rangle_{\mathcal{E}_S} \\ &= \int\limits_{[-1,1]^N} f_{(s_1,...,s_N)}(\lambda_1, \ldots, \lambda_N) P^{(\mathcal{E}_S)}_{(s_1,...,s_N)}(\mathrm{d}\lambda_1 \times \cdots \times \mathrm{d}\lambda_N) \end{aligned} \tag{4}$$

of the most general form, specified for each N-partite joint measurement $(s_1, ..., s_N)$ by a bounded real-valued function $f_{(s_1,...,s_N)}$ of outcomes $(\lambda_1, \ldots, \lambda_N) \in [-1,1]^N$ at all N sites. Each linear combination (3) is specified by a family $\Phi_S = \{f_{(s_1,...,s_N)}\}$ of these functions.

Depending on a choice of a function $f_{s_1,...,s_N}$, an average (4) may refer either to the joint probability of events observed under this joint measurement at $M \leq N$ sites or to the expectation

$$\langle \lambda_1^{(s_1)} \cdot \ldots \cdot \lambda_{n_M}^{(s_{n_M})} \rangle_{\mathcal{E}_S} = \int\limits_{[-1,1]^N} \lambda_1 \cdot \ldots \cdot \lambda_{n_M} P^{(\mathcal{E}_S)}_{(s_1,...,s_N)}(\mathrm{d}\lambda_1 \times \cdots \times \mathrm{d}\lambda_N) \tag{5}$$

of the product of outcomes observed at $M \leq N$ sites or may have a more complicated form. In quantum information, the product expectation (5) is referred to as a correlation function.